

NAG Fortran Library Routine Document

H02BZF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

Warning. The specification of the parameter LIWORK changed at Mark 16: the minimum dimension of the array IWORK has been increased by $N + 3$.

1 Purpose

H02BZF extracts more information associated with the solution of an integer programming problem computed by H02BBF.

2 Specification

```

SUBROUTINE H02BZF(N, M, BL, BU, CLAMDA, ISTATE, IWORK, LIWORK, RWORK,
1                LRWORK, IFAIL)
  INTEGER        N, M, ISTATE(N+M), IWORK(LIWORK), LIWORK, LRWORK,
1                IFAIL
  real         BL(N+M), BU(N+M), CLAMDA(N+M), RWORK(LRWORK)

```

3 Description

H02BZF extracts the following information associated with the solution of an integer programming problem computed by H02BBF. The upper and lower bounds used for the solution, the Lagrange multipliers (costs), and the status of the variables at the solution.

In the branch and bound method employed by H02BBF, the arrays BL and BU are used to impose restrictions on the values of the integer variables in each sub-problem. That is, if the variable x_j is restricted to take value v_j in a particular sub-problem, then $BL(j) = BU(j) = v_j$ is set in the sub-problem. Thus, on exit from this routine, some of the elements of BL and BU which correspond to integer variables may contain these imposed values, rather than those originally supplied to H02BBF.

4 References

None.

5 Parameters

- 1: N – INTEGER *Input*
On entry: this **must** be the same parameter N as supplied to H02BBF.
Constraint: $N > 0$.
- 2: M – INTEGER *Input*
On entry: this **must** be the same parameter M as supplied to H02BBF.
Constraint: $M \geq 0$.
- 3: BL(N+M) – *real* array *Output*
On exit: if H02BBF exits with IFAIL = 0, 7 or 9, the values in the array BL contain the lower bounds imposed on the integer solution for all the constraints. The first N elements contain the lower bounds on the variables, and the next M elements contain the lower bounds for the general linear constraints (if any).

- 4: BU(N+M) – *real* array *Output*
On exit: if H02BBF exits with IFAIL = 0, 7 or 9, the values in the array BU contain the upper bounds imposed on the integer solution for all the constraints. The first N elements contain the upper bounds on the variables, and the next M elements contain the upper bounds for the general linear constraints (if any).
- 5: CLAMDA(N+M) – *real* array *Output*
On exit: if H02BBF exits with IFAIL = 0, 7 or 9, the values in the array CLAMDA contain the values of the Lagrange multipliers for each constraint with respect to the current working set. The first N elements contain the multipliers (reduced costs) for the bound constraints on the variables, and the next M elements contain the multipliers (shadow costs) for the general linear constraints (if any).
- 6: ISTATE(N+M) – INTEGER array *Output*
On exit: if H02BBF exits with IFAIL = 0, 7 or 9, the values in the array ISTATE indicate the status of the constraints in the working set at an integer solution. Otherwise, ISTATE indicates the composition of the working set at the final iterate. The significance of each possible value of ISTATE(*j*) is as follows.
- | ISTATE(<i>j</i>) | Meaning |
|--------------------|---|
| –2 | The constraint violates its lower bound by more than TOLFES (the feasibility tolerance, see H02BBF). |
| –1 | The constraint violates its upper bound by more than TOLFES. |
| 0 | The constraint is satisfied to within TOLFES, but is not in the working set. |
| 1 | This inequality constraint is included in the working set at its lower bound. |
| 2 | This inequality constraint is included in the working set at its upper bound. |
| 3 | This constraint is included in the working set as an equality. This value of ISTATE can occur only when $BL(j) = BU(j)$. |
| 4 | This corresponds to an integer solution being declared with x_j being temporarily fixed at its current value. This value of ISTATE can occur only when IFAIL = 0, 7 or 9 on exit from H02BBF. |
- 7: IWORK(LIWORK) – INTEGER array *Workspace*
This **must** be the same parameter IWORK as supplied to H02BBF. It is used to pass information from H02BBF to H02BZF and therefore the contents of this array **must not** be changed before calling H02BZF.
- 8: LIWORK – INTEGER *Input*
On entry: the dimension of the array IWORK as declared in the (sub)program from which H02BZF is called.
- 9: RWORK(LRWORK) – *real* array *Workspace*
This **must** be the same parameter RWORK as supplied to H02BBF. It is used to pass information from H02BBF to H02BZF and therefore the contents of this array **must not** be changed before calling H02BZF.
- 10: LRWORK – INTEGER *Input*
On entry: the dimension of the array RWORK as declared in the (sub)program from which H02BZF is called.
- 11: IFAIL – INTEGER *Input/Output*
On entry: IFAIL must be set to 0, –1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.
On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0 . **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry $IFAIL = 0$ or -1 , explanatory error messages are output on the current error message unit (as defined by $X04AAF$).

Errors or warnings detected by the routine:

$IFAIL = 1$

On entry, $N \leq 0$,
or $M < 0$.

7 Accuracy

Not applicable.

8 Further Comments

None.

9 Example

One of the applications of integer programming is to the so-called diet problem. Given the nutritional content of a selection of foods, the cost of each food, the amount available of each food and the consumer's minimum daily nutritional requirements, the problem is to find the cheapest combination. This gives rise to the following problem:

minimize

$$c^T x$$

subject to

$$Ax \geq b, 0 \leq x \leq u,$$

where

$$c = (3 \quad 24 \quad 13 \quad 9 \quad 2019)^T, x = (x_1, x_2, x_3, x_4, x_5, x_6)^T$$

is integer,

$$A = \begin{pmatrix} 110 & 205 & 160 & 160 & 420 & 260 \\ 4 & 32 & 13 & 8 & 4 & 14 \\ 2 & 12 & 54 & 285 & 22 & 80 \end{pmatrix}, \quad b = \begin{pmatrix} 2000 \\ 55 \\ 800 \end{pmatrix}$$

and

$$u = (4 \quad 3 \quad 2 \quad 8 \quad 2 \quad 2)^T.$$

The rows of A correspond to energy, protein and calcium and the columns of A correspond to oatmeal, chicken, eggs, milk, pie and bacon respectively.

The following program solves the above problem to obtain the optimal integer solution and then examines the effect of increasing the energy required to 2200 units.

9.1 Program Text

Note: the listing of the example program presented below uses *bold italicised* terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```

*      H02BZF Example Program Text
*      Mark 16 Revised. NAG Copyright 1993.
*      .. Parameters ..
INTEGER          NIN, NOUT
PARAMETER       (NIN=5,NOUT=6)
INTEGER          NMAX, MMAX
PARAMETER       (NMAX=10,MMAX=10)
INTEGER          LDA
PARAMETER       (LDA=MMAX)
INTEGER          LIWORK, LRWORK
PARAMETER       (LIWORK=1000,LRWORK=1000)
*      .. Local Scalars ..
real           BIGBND, INIVAL, OBJMIP, TOLFES, TOLIV
INTEGER          I, IFAIL, INTFST, ITMAX, J, M, MAXDPT, MAXNOD,
+              MSGLVL, N
*      .. Local Arrays ..
real           A(LDA,NMAX), BL(NMAX+MMAX), BU(NMAX+MMAX),
+              CLAMDA(NMAX+MMAX), CVEC(NMAX), RWORK(LRWORK),
+              X(NMAX)
INTEGER          INTVAR(NMAX), ISTATE(NMAX+MMAX), IWORK(LIWORK)
CHARACTER*8     NAMES(NMAX+MMAX)
*      .. External Subroutines ..
EXTERNAL        H02BBF, H02BZF, OUTSOL
*      .. Executable Statements ..
WRITE (NOUT,*) 'H02BZF Example Program Results'
*      Skip heading in data file
READ (NIN,*)
READ (NIN,*) N, M
IF (N.LE.NMAX .AND. M.LE.MMAX) THEN
*
*          Read ITMAX, MSGLVL, MAXNOD, INTFST, MAXDPT, TOLFES, TOLIV,
*          CVEC, A, BIGBND, BL, BU, INTVAR and X from data file
*
      READ (NIN,*) ITMAX, MSGLVL
      READ (NIN,*) MAXNOD
      READ (NIN,*) INTFST, MAXDPT
      READ (NIN,*) TOLFES, TOLIV
      READ (NIN,*) (CVEC(J),J=1,N)
      READ (NIN,*) (NAMES(J), (A(I,J),I=1,M),J=1,N)
      READ (NIN,*) BIGBND
      READ (NIN,*) (BL(I),I=1,N)
      READ (NIN,*) (NAMES(N+I),BL(N+I),I=1,M)
      READ (NIN,*) (BU(I),I=1,N+M)
      READ (NIN,*) (INTVAR(I),I=1,N)
      READ (NIN,*) (X(I),I=1,N)
*
*          Solve the IP problem using H02BBF
*
      IFAIL = -1
*
      CALL H02BBF(ITMAX,MSGLVL,N,M,A,LDA,BL,BU,INTVAR,CVEC,MAXNOD,
+              INTFST,MAXDPT,TOLIV,TOLFES,BIGBND,X,OBJMIP,IWORK,
+              LIWORK,RWORK,LRWORK,IFAIL)
*
      IF (IFAIL.EQ.0 .OR. IFAIL.EQ.7 .OR. IFAIL.EQ.9) THEN
          WRITE (NOUT,99999) 'IP objective value = ', OBJMIP
*
*          Get information about the solution
*
          IFAIL = 0
*
          CALL H02BZF(N,M,BL,BU,CLAMDA,ISTATE,IWORK,LIWORK,RWORK,
+              LRWORK,IFAIL)
*
*          Print the solution

```

```

*
+       CALL OUTSOL(N,M,A,LDA,BL,BU,X,ISTATE,CLAMDA,BIGBND,NAMES,
+           NOUT)
*
*       Increase the energy requirements and solve the modified IP
*       problem using the current IP solution as the starting point
*
+       INIVAL = BL(N+1)
+       READ (NIN,*) BL(N+1)
+       WRITE (NOUT,99998) 'Increase the energy requirements from',
+           INIVAL, 'to', BL(N+1)
*
+       IFAIL = -1
*
+       CALL H02BBF(ITMAX,MSGLVL,N,M,A,LDA,BL,BU,INTVAR,CVEC,MAXNOD,
+           INTFST,MAXDPT,TOLIV,TOLFES,BIGBND,X,OBJMIP,
+           IWORK,LIWORK,RWORK,LRWORK,IFAIL)
*
+       IF (IFAIL.EQ.0 .OR. IFAIL.EQ.7 .OR. IFAIL.EQ.9) THEN
+           WRITE (NOUT,99999) 'IP objective value = ', OBJMIP
*
*       Get information about the solution
*
+       IFAIL = 0
*
+       CALL H02BZF(N,M,BL,BU,CLAMDA,ISTATE,IWORK,LIWORK,RWORK,
+           LRWORK,IFAIL)
*
*       Print the solution
*
+       CALL OUTSOL(N,M,A,LDA,BL,BU,X,ISTATE,CLAMDA,BIGBND,NAMES,
+           NOUT)
*
+       ELSE
+           WRITE (NOUT,99997) ' H02BBF terminated with IFAIL = ',
+               IFAIL
+       END IF
+       ELSE
+           WRITE (NOUT,99997) ' H02BBF terminated with IFAIL = ', IFAIL
+       END IF
+       END IF
+       STOP
*
99999 FORMAT (//1X,A,1P,G16.4)
99998 FORMAT (//1X,A,2X,1P,G10.4,2X,A,2X,1P,G10.4)
99997 FORMAT (1X,A,I3)
END
SUBROUTINE OUTSOL(N,M,A,LDA,BL,BU,X,ISTATE,CLAMDA,BIGBND,NAMES,
+   NOUT)
*
+   .. Scalar Arguments ..
+   real           BIGBND
+   INTEGER        LDA, M, N, NOUT
*
+   .. Array Arguments ..
+   real         A(LDA,*), BL(N+M), BU(N+M), CLAMDA(N+M), X(N)
+   INTEGER        ISTATE(N+M)
+   CHARACTER*8    NAMES(N+M)
*
+   .. Local Scalars ..
+   real         B1, B2, RES, RES2, V, WLAM
+   INTEGER        IS, J, K
+   CHARACTER*80   REC
*
+   .. Local Arrays ..
+   CHARACTER*2    LSTATE(-2:4)
*
+   .. External Functions ..
+   real         sdot
+   EXTERNAL       sdot
*
+   .. Intrinsic Functions ..
+   INTRINSIC     ABS
*
+   .. Data statements ..
+   DATA         LSTATE(-2)/'--'/, LSTATE(-1)/'++'/,
+   +             LSTATE(0)/'FR'/, LSTATE(1)/'LL'/,
+   +             LSTATE(2)/'UL'/, LSTATE(3)/'EQ'/,

```

```

+           LSTATE(4)/'TF'/
* .. Executable Statements ..
*
WRITE (NOUT,99999)
DO 20 J = 1, N + M
  B1 = BL(J)
  B2 = BU(J)
  WLAM = CLAMDA(J)
  IS = ISTATE(J)
  IF (J.LE.N) THEN
*     The variables x.
    K = J
    V = X(J)
  ELSE
*     The linear constraints A*x.
    IF (J.EQ.N+1) WRITE (NOUT,99998)
    K = J - N
    V = sdot(N,A(K,1),LDA,X,1)
  END IF
*
*     Print a line for the j-th variable or constraint.
*
  RES = V - B1
  RES2 = B2 - V
  IF (ABS(RES).GT.ABS(RES2)) RES = RES2
  WRITE (REC,99997) NAMES(J), LSTATE(IS), V, B1, B2, WLAM, RES
  IF (B1.LE.-BIGBND) REC(29:42) = ' None '
  IF (B2.GE.BIGBND) REC(43:56) = ' None '
  WRITE (NOUT,'(A)') REC
20 CONTINUE
RETURN
*
99999 FORMAT (//1X,'Varbl',3X,'State',5X,'Value',5X,'Lower Bound',3X,
+           'Upper Bound',4X,'Lagr Mult',3X,'Residual',/)
99998 FORMAT (//1X,'L Con',3X,'State',5X,'Value',5X,'Lower Bound',3X,
+           'Upper Bound',4X,'Lagr Mult',3X,'Residual',/)
99997 FORMAT (1X,A8,2X,A2,1X,1P,3G14.4,1P,G12.4,1P,G12.4)
END

```

9.2 Program Data

H02BZF Example Program Data

```

6 3                               :Values of N and M
0 0                               :Values of ITMAX and MSGGLVL
0                                 :Value of MAXNOD
0 9                               :Values of INTFST and MAXDPT
0.0 0.0                          :Values of TOLFES and TOLIV
3.0 24.0 13.0 9.0 20.0 19.0     :End of CVEC
'Oatmeal' 110.0 4.0 2.0
'Chicken' 205.0 32.0 12.0
'Eggs' 160.0 13.0 54.0
'Milk' 160.0 8.0 285.0
'Pie' 420.0 4.0 22.0
'Bacon' 260.0 14.0 80.0
1.0E+20                               :End of matrix A
0.0 0.0 0.0 0.0 0.0             :Value of BIGBND
'Energy' 2000.0 'Protein' 55.0 'Calcium' 800.0 :End of BL
4.0 3.0 2.0 8.0 2.0 2.0 1.0E+20 1.0E+20 1.0E+20 :End of BU
1 1 1 1 1 1                       :End of INTVAR
0.0 0.0 0.0 0.0 0.0 0.0         :End of X
2200.0                             :Change 'Energy' in RHS

```

9.3 Program Results

H02BZF Example Program Results

IP objective value = 97.00

Varbl	State	Value	Lower Bound	Upper Bound	Lagr Mult	Residual
Oatmeal	EQ	4.000	4.000	4.000	3.000	0.000
Chicken	LL	0.000	0.000	3.000	24.00	0.000
Eggs	LL	0.000	0.000	2.000	13.00	0.000
Milk	LL	5.000	5.000	8.000	9.000	0.000
Pie	EQ	2.000	2.000	2.000	20.00	0.000
Bacon	LL	0.000	0.000	2.000	19.00	0.000

L Con	State	Value	Lower Bound	Upper Bound	Lagr Mult	Residual
Energy	FR	2080.	2000.	None	0.000	80.00
Protein	FR	64.00	55.00	None	0.000	9.000
Calcium	FR	1477.	800.0	None	0.000	677.0

Increase the energy requirements from 2000. to 2200.

IP objective value = 106.0

Varbl	State	Value	Lower Bound	Upper Bound	Lagr Mult	Residual
Oatmeal	EQ	4.000	4.000	4.000	3.000	0.000
Chicken	LL	0.000	0.000	3.000	24.00	0.000
Eggs	LL	0.000	0.000	2.000	13.00	0.000
Milk	LL	6.000	6.000	8.000	9.000	0.000
Pie	EQ	2.000	2.000	2.000	20.00	0.000
Bacon	LL	0.000	0.000	2.000	19.00	0.000

L Con	State	Value	Lower Bound	Upper Bound	Lagr Mult	Residual
Energy	FR	2240.	2200.	None	0.000	40.00
Protein	FR	72.00	55.00	None	0.000	17.00
Calcium	FR	1762.	800.0	None	0.000	962.0
